Credit Default Swap Spreads and Systemic Financial Risk

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Introduction

- *What is the joint probability of default of large financial institutions?*
- Systemic default risk:

$$Pr\{at\ least\ r\ LFIs\ default\}$$

  - Banks are interconnected and exposed to common shocks
  - Defaults not independent even at short horizons
  - Severe consequences of multiple defaults

Introduction: this paper

- Difficult to measure
  - Rare events
  - Prices that reflect *individual* defaults (bonds,CDSs) but not multiple defaults
  - Traditional measures: aggregate individual risks
- In this paper
  1. Exploit counterparty risk to learn about $P(A_i \cap A_j)$: enrich information set
  2. Derive tightest bounds on multiple default risk

CDS and counterparty risk

- Credit Default Swap is an OTC contract designed to transfer the credit risk of the *reference entity*
- Counterparty risk in CDSs: if seller defaults, contract terminates
- "Double default" relevant for pricing: discount relative to the corresponding bond
- Bond issued by i: $P(A_i)$. CDS by j on i: $P(A_i) - P(A_i \cap A_j)$

CDS and counterparty risk

- Collateral
  - static: very costly -> dynamic
  - Not widely used with dealers (66% of contracts in 2008)
  - When margin set to current exposure, subject to jumps
  - Collateral can be less than current exposure (Goldman)
  - Buyers aware of counterparty risk (CDS against seller)

Theory

- Assume we observe

$$p_i \, : \, P(A_i)$$

$$z_{ji} \, : \, P(A_i), P(A_i \cap A_j)$$

- Look for P_r: $P\{\textit{at least r default}\}$ (*information of order N - systemic)*

$$P_1 = P(A_1 \cup A_2 \cup A_3)$$

$$P_2 = P((A_1 \cap A_2) \cup (A_2 \cap A_3) \cup (A_1 \cap A_3))$$

$$P_3 = P(A_1 \cap A_2 \cap A_3)$$

Theory

- Becomes

$max\, Pr\{at\ least\ r\ default\}$	$max_p\, c_r' p$
$P(A_i) = a_i$ $P(A_i \cap A_j) = a_{ij}$	$Ap = b$
Consistent probability system	$p \geq 0$ $i'p = 1$

Implementation

- Assume a simple, discretized pricing model for bonds and CDSs
- Constant hazard rates
- If two banks default in the same month -> double default
- Assume recovery rates R=30%, S=30%
- The price of a bond depends both on $P(A_i)$ and on liquidity γ_t^i (transaction cost, cost of capital).

Implementation

- Impose a *lower bound* for the liquidity process γ_i of bonds
- Calibrating $\gamma_t^i \geq \underline{\gamma}_t^i$, obtain

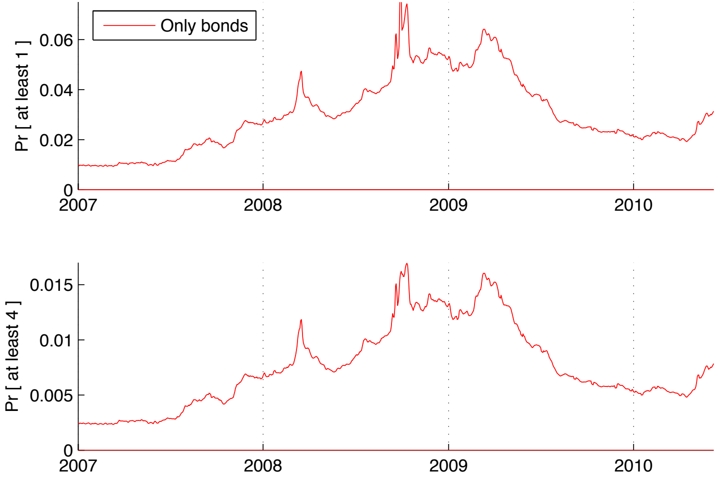
$$P(A_i) \leq h_i(\underline{\gamma}_t^i)$$

  - Nonnegative
  - Calibrated to 2004
  - Calibrated to nonfinancial firms

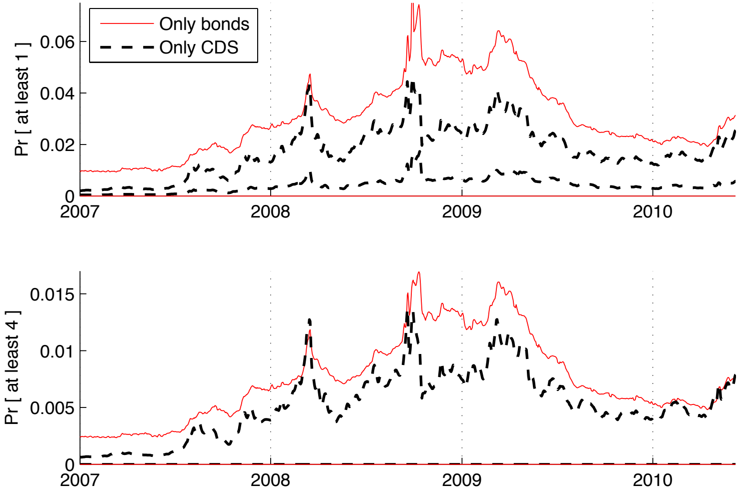
Implementation

- Observe average CDS spreads:
  - z_{ji} linear function of $P(A_i)$ and $P(A_i \cap A_j)$
  - $\bar{z}_i$ linear function of $P(A_i)$ and $\frac{1}{N-1}\sum_{j \neq i} P(A_i \cap A_j)$
  - One constraint for each i

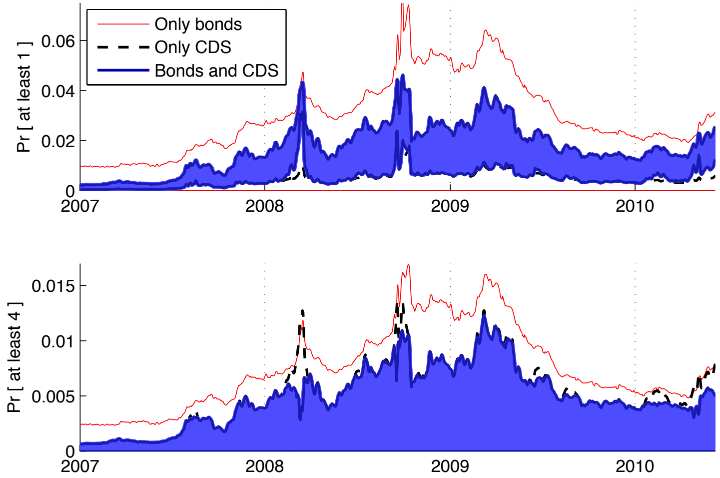
Systemic risk - $\gamma_t^i \geq 0$



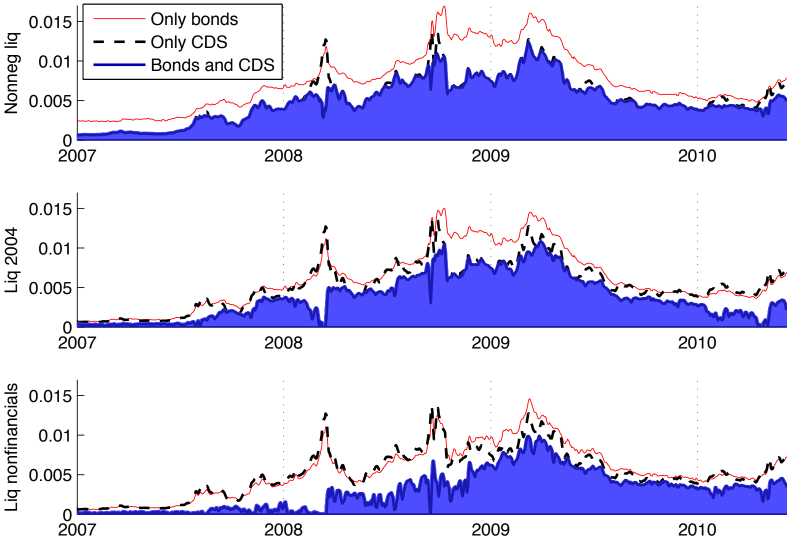
Systemic risk - $\gamma_t^i \geq 0$



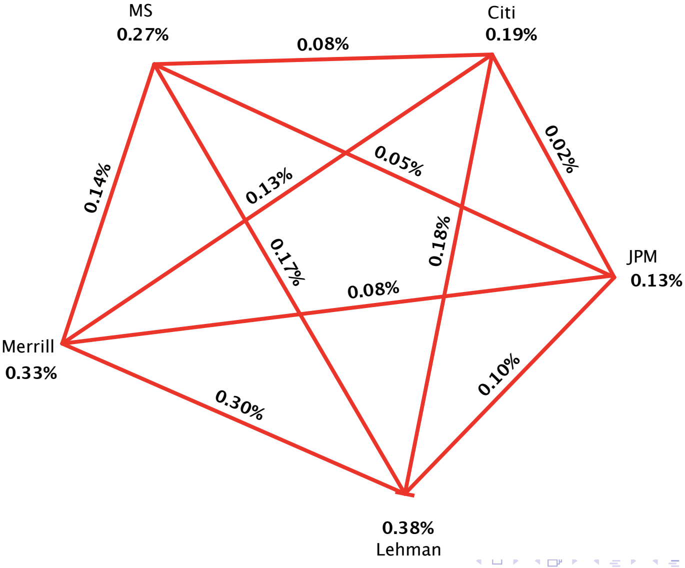
Systemic risk - $\gamma_t^i \geq 0$



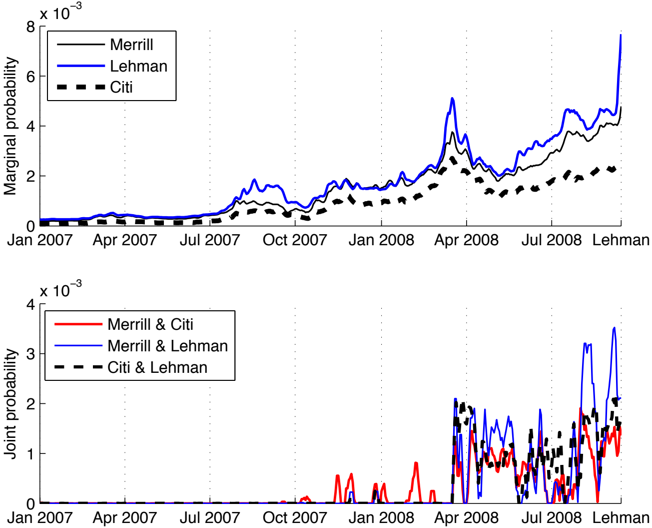
Systemic risk measures: assumptions on liquidity



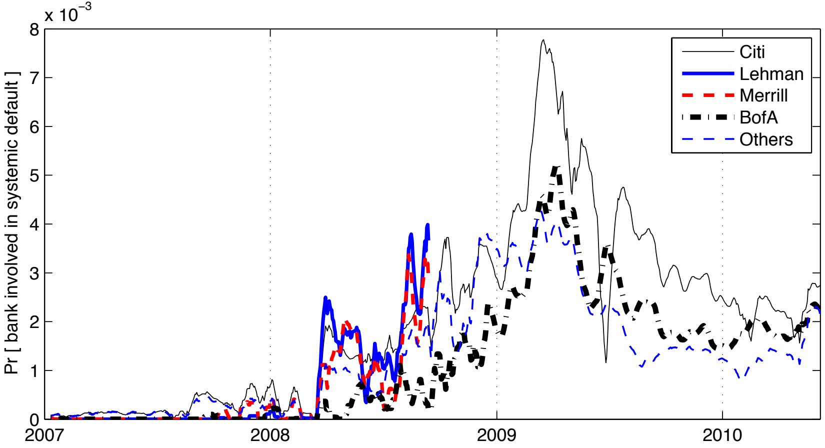
Picture of network 8/4/2008



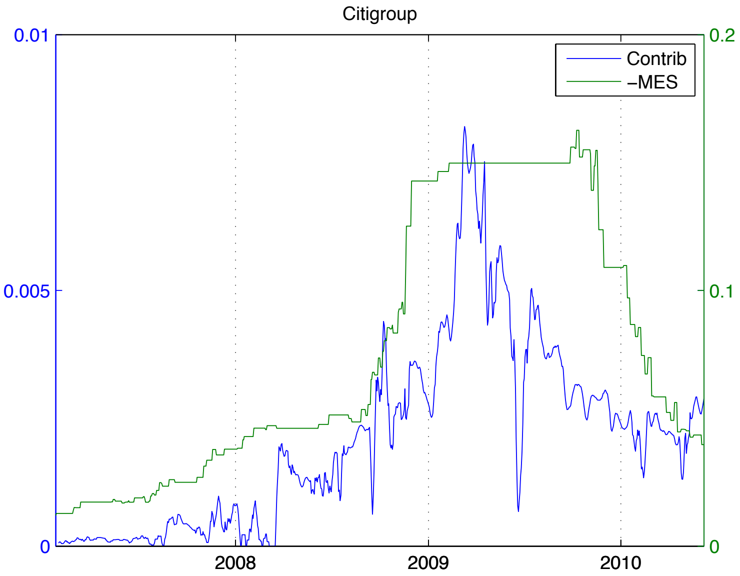
Marginal and pairwise probabilities



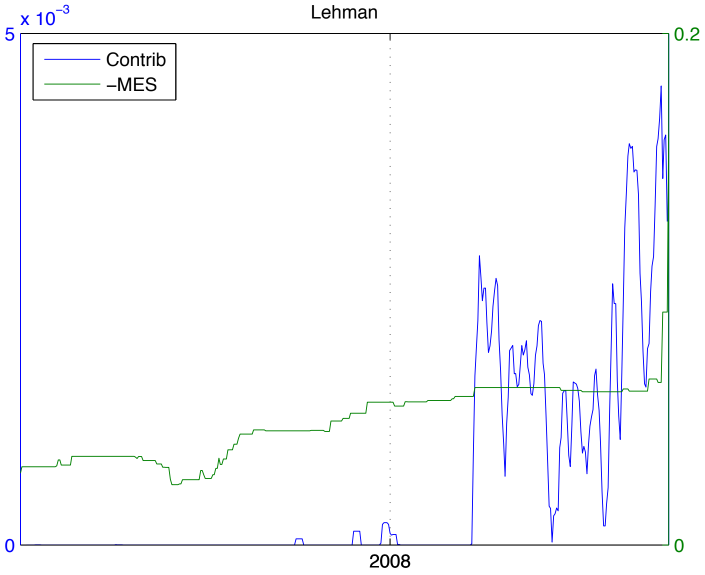
Contribution: $Pr\{at\, least\, 4 \cap j\}$



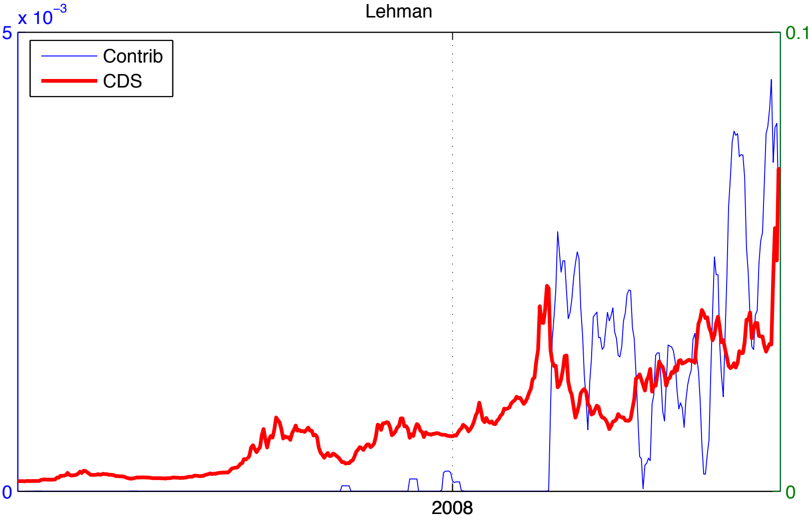
Explore: Contrib vs. MES



Explore: Contrib vs. MES



Explore: Contrib vs. MES



Conclusion

- Taking counterparty risk into account, CDS provide information on pairwise default probabilities
- These can be optimally aggregated across the financial network: LP bounds
- The optimal bounds are tight under assumptions on liquidity and allow to distinguish idiosyncratic/systemic risk
- Also learn about the contribution of each institution

Conclusion

- We learn that systemic risk really started to increase in late 2008:
  - *If systemic risk was so high in January-March 2008, why did the average CDS spread go up so much?*
  - *Why were people so keen to buy insurance from unreliable counterparties?*
- Things to explore
  - Correlation of contribution to systemic risk with other measures (MES, CoVar, stress test)
  - Pairwise default risk and correlation of equity returns
  - Systemic risk and puts

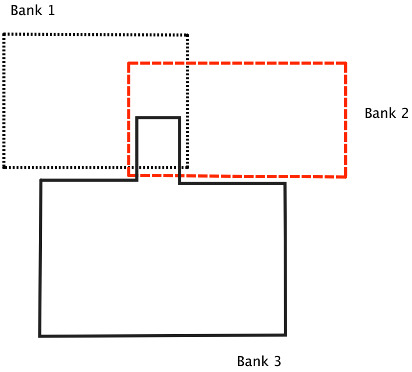
Extra Slides

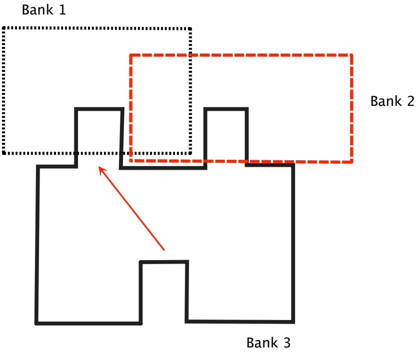
- Additional details
  - Simple Example of Bounds
  - Linear Programming Algorithm
  - Implementation
  - Pricing Formulas
  - Data
  - Symmetry

Extra Slides

- Robustness to assumptions on recovery rates:
  - Robustness to R and S
  - Stochastic and Time Varying Recovery Rates
- All other robustness tests:
  - Robustness Results
  - Derivations
- References:
  - Arora et al.

Theory: simple example





Theory: simple example

- Suppose we observe, from bonds:

$$P(A_1) = P(A_2) = P(A_3) = 0.2$$

- From CDSs:

$$P(A_1 \cap A_2) = P(A_2 \cap A_3) = 0.07$$

$$P(A_1 \cap A_3) = 0.01$$

- Tightest bounds

$$0.45 \leq P_1 \leq 0.46$$

$$0.13 \leq P_2 \leq 0.15$$

$$0 \leq P_3 \leq 0.01$$

Theory: simple example

- **Heterogeneity**. Suppose still

$$P(A_1) = P(A_2) = P(A_3) = 0.2$$

but now we only know

$$\frac{P(A_1 \cap A_2) + P(A_2 \cap A_3) + P(A_1 \cap A_3)}{3} = 0.05$$

- Then

Full information	*Only average*
$0.45 \leq P_1 \leq 0.46$	$0.45 \leq P_1 \leq 0.50$
$0.13 \leq P_2 \leq 0.15$	$0.05 \leq P_2 \leq 0.15$
$0 \leq P_3 \leq 0.01$	$0 \leq P_3 \leq 0.05$

Linear Programming Algorithm

- Start from problem:

$$max\, P_r$$

s.t.

$$P(A_i) = a_i$$

$$...$$

$$P(A_i \cap A_j) = a_{ij}$$

Linear Programming Algorithm

- Obtain:

$$max_p \, c_r' p$$

s.t.

$$p \geq 0$$

$$i' p = 1$$

$$Ap = b$$

Linear Programming Algorithm

- Start with matrix B $(2^N, N)$
- Rows are binary representation of $0 \ldots 2^N - 1$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ & \ldots & & \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

- Each row is event of type

$$A_1^* \cap A_2^* \cap \ldots \cap A_N^*$$

- where $A_j^* = A_j$ if element j of the row is 1, and $A_j^* = \overline{A}_j$ otherwise

Linear Programming Algorithm

- p contains probabilities of these events

$$p \geq 0$$

$$p' i = 1$$

- $P(A_i)$:

$$P(A_i) = \sum_{j:B(j,i)=1} p_j$$

or

$$P(A_i) = a^{i'} p$$

$$a^i_j = B(j,i)$$

Linear Programming Algorithm

- $P(A_i \cap A_k)$:

$$P(A_i \cap A_k) = \sum_{j: B(j,i)=1 \, and \, B(j,k)=1} p_j$$

or:

$$P(A_i \cap A_k) = b^{ik'} p$$

for a vector b_{ik} of size $(2^N, 1)$ s.t.:

$$b^{ik}_j = B(j,i)B(j,k)$$

Linear Programming Algorithm

- P_r:

$$P_r = \sum_{j:(\sum_{h=1:N} B(j,h))\geq r} p_j$$

or

$$P_r = c^{r'} p$$

for a vector c^r of size $(2^N, 1)$ s.t.:

$$c_j^r = I\left[\sum_{h=1:N} B(j,h) \geq r\right]$$

Implementation: 1 - Pricing

- Contracts span long horizons
- Contracts priced again every time t looking forward, assuming
  - Constant hazard rates h_t^i
  - Constant bond liquidity premium γ_t^i
- Discretize by month
- Joint default in a month <=> double default
- Seller default and reference survives until next month -> small change in reference risk
- Recovery R=30%, S=30%

Implementation: 1 - Pricing (bonds)

$$B^{ij}(t, T^{ij}) = c^{ij}\left(\sum_{s=t+1}^{T^{ij}} \delta(t,s)(1-h_t^i)^{s-t}(1-\gamma_t^i)^{s-t}\right) +$$

$$+\delta(t, T^{ij})(1-h_t^i)^{T^{ij}-t}(1-\gamma_t^i)^{T^{ij}-t}$$

$$+R\left(\sum_{s=t+1}^{T^{ij}} \delta(t,s)(1-h_t^i)^{s-t-1}(1-\gamma_t^i)^{s-t-1}h_t^i\right)$$

Implementation: 1 - Pricing (bonds)

- Bond liquidity: constant conveniency yield γ_t^i
- Interpretation. Garleanu and Pedersen (2010):

$$E_t[R_{t+1}^{ij} - R_{t+1}^f] = -\frac{Cov_t(M_{t+1}, R_{t+1}^{ij} - R_{t+1}^f)}{E_t[M_{t+1}]} + m_t^i x_t \psi_t$$

  - m_t^i margin for senior unsecured bonds of firm i
  - x_t proportion of agents constrained
  - ψ_t shadow cost of capital

$$\gamma_t^i \approx m_t^i x_t \psi_t = \alpha^i \lambda_t$$

Implementation: 1 - Pricing (CDSs)

$$\sum_{s=t}^{T-1} \delta(t,s)(1-P(A_i \cup A_j))^{s-t} z_{ji} =$$

$$= \sum_{s=t+1}^{T} \delta(t,s)(1-P(A_i \cup A_j))^{s-t-1}$$

$$\left\{ \left[P(A_i) - P(A_i \cap A_j) \right] (1-R) + S \left[P(A_i \cap A_j) \right] (1-R) \right\}$$

Implementation: 2 - Calibration of liquidity γ_t^i

- Bond liquidity: constant conveniency yield $\gamma_t^i = \alpha^i \lambda_t$
  - λ_t: common variations in margins, cost of capital, constrained agents
- Calibrating $\gamma_t^i \geq \underline{\gamma}_t^i$, obtain

$$P(A_i) \leq h_i(\underline{\gamma}_t^i)$$

1. $\gamma_t^i \geq 0$
2. $\gamma_t^i \geq \alpha^i$: liquidity at least as of 2004

Implementation: 2 - Calibration of liquidity γ_t^i

3. For a group K of A-rated (or better) nonfinancial firms

- double default risk is low
- calibrate matching the bond-CDS basis

$$\gamma_t^k = \alpha^k \lambda_t^*$$

- and assume that for financials

$$\gamma_t^i \geq \alpha^i \lambda_t^*$$

Implementation: 3 - Availability of CDS spreads

- Observe average CDS spreads:
  - z_{ji} linear function of $P(A_i)$ and $P(A_i \cap A_j)$
  - $\bar{z}_i$ linear function of $P(A_i)$ and $\frac{1}{N-1}\sum_{j \neq i} P(A_i \cap A_j)$
  - One constraint for each i
- Do not observe contributors of Markit quotes
  - Pick 15 dealers covering 90% of CDS market

Pricing formulas: bonds

Bonds:

$$B^{ij}(t, T^{ij}) = c^{ij}\left(\sum_{s=t+1}^{T^{ij}} \delta(t,s)(1-h_t^i)^{s-t}(1-\gamma_t^i)^{s-t}\right)+$$

$$+\delta(t, T^{ij})(1-h_t^i)^{T^{ij}-t}(1-\gamma_t^i)^{T^{ij}-t}$$

$$+R\left(\sum_{s=t+1}^{T^{ij}} \delta(t,s)(1-h_t^i)^{s-t-1}(1-\gamma_t^i)^{s-t-1}h_t^i\right)$$

Pricing formulas: CDSs

$$\sum_{s=t}^{T-1} \delta(t,s)(1-P(A_i \cup A_j))^{s-t} z_{ji} =$$

$$= \sum_{s=t+1}^{T} \delta(t,s)(1-P(A_i \cup A_j))^{s-t-1}$$

$$\left\{ \left[P(A_i) - P(A_i \cap A_j) \right] (1-R) + S \left[P(A_i \cap A_j) \right] (1-R) \right\}$$

Pricing formulas: CDSs

Linearize to use as a constraint:

$$z_{ji,t} = (P(A_i) - (1-S)P(A_i \cap A_j)) \frac{\left[\sum_{s=t+1}^{T} \delta(t,s)\right](1-R)}{\left[\sum_{s=t}^{T-1} \delta(t,s)\right]}$$

Data

- Bonds
  - Look on Bloomberg and Markit for all bonds that are issued by institution i
  - Restrict to senior unsecured fixed or zero coupon: no callable, putable, sinkable, structured
  - TRACE-eligible bonds: use TRACE closing price
  - Other bonds: generic closing price

Data

- Risk-free rate: zero-coupon government bonds
- CDS: Markit
- Period: 2004 to June 2010

Data

Table 1

	Avg valid bonds	2004	2005	2006	2007	2008	2009	2010
Abn Amro	3.3	1.8	2.2	4.0	4.4	3.0	3.5	5.3
Bank of America	32.3	17.5	25.4	29.3	32.9	35.3	41.8	55.8
Barclays	14.8	3.1	3.0	2.4	2.5	9.1	38.5	78.0
Bear Stearns	11.4	7.2	9.8	12.6	15.3	15.4	-	-
Bnp Paribas	7.0	0.5	2.0	3.0	3.9	6.7	18.0	22.6
Citigroup	36.5	21.6	24.3	31.7	40.0	43.2	49.5	54.5
Credit Suisse	5.4	1.9	2.3	2.8	2.7	5.0	11.6	17.4
Deutsche Bank	42.1	5.3	10.4	42.3	68.9	54.4	58.8	67.9
Goldman Sachs	39.6	19.3	26.1	34.3	40.4	49.0	57.0	63.0
JP Morgan	17.3	6.6	11.1	14.0	17.4	22.2	27.4	27.9
Lehman Brothers	20.1	10.5	15.2	20.5	26.5	31.4	-	-
Merrill Lynch	35.7	22.7	33.0	38.4	43.0	44.0	-	-
Morgan Stanley	25.5	12.5	14.6	17.5	22.2	30.0	45.0	49.0
UBS	8.2	0.3	0.7	1.0	3.1	8.3	22.4	36.6
Wachovia	6.1	2.9	3.5	5.7	7.4	9.1	7.7	7.3

Note: first column reports average number of bonds for each institution that are used for the estimation of marginal default probabilities. Columns 2-8 break this number down by year.

Data

Table 2

	Avg CDS spread	Std CDS spread	Min spread	Max spread
Abn Amro	45.8	46.1	5.0	190.5
Bank of America	66.5	71.7	7.4	390.7
Barclays	54.3	60.0	5.5	261.9
Bear Stearns	54.2	69.7	18.0	736.9
Bnp Paribas	33.8	32.2	5.4	163.9
Citigroup	100.4	129.7	6.5	638.3
Credit Suisse	53.0	51.3	9.0	261.4
Deutsche Bank	49.8	45.1	8.9	190.0
Goldman Sachs	84.2	86.4	17.2	579.3
JP Morgan	53.1	42.8	10.9	227.3
Lehman Brothers	70.7	86.9	18.0	701.7
Merrill Lynch	59.9	71.9	14.4	447.7
Morgan Stanley	112.5	144.2	16.6	1385.6
UBS	59.3	72.4	4.2	357.2
Wachovia	73.9	93.5	9.3	1487.7

Data

Table 2

	Avg basis	Std basis	Min basis	Max basis
Abn Amro	-46.2	44.4	-248.2	34.6
Bank of America	-71.9	64.8	-412.5	217.5
Barclays	-41.2	60.1	-324.8	111.6
Bear Stearns	-53.6	24.6	-298.0	40.6
Bnp Paribas	-53.9	49.7	-321.8	86.8
Citigroup	-76.5	88.1	-804.6	59.2
Credit Suisse	-50.5	44.0	-276.6	52.6
Deutsche Bank	-24.4	27.6	-174.2	65.2
Goldman Sachs	-79.0	91.3	-502.4	75.1
JP Morgan	-76.5	57.9	-322.2	32.1
Lehman Brothers	-61.9	44.5	-540.0	10.9
Merrill Lynch	-51.7	40.4	-200.0	26.2
Morgan Stanley	-82.6	107.3	-1256.5	223.2
UBS	-65.4	58.5	-343.5	34.2
Wachovia	-87.2	120.1	-2509.8	88.0

Symmetry

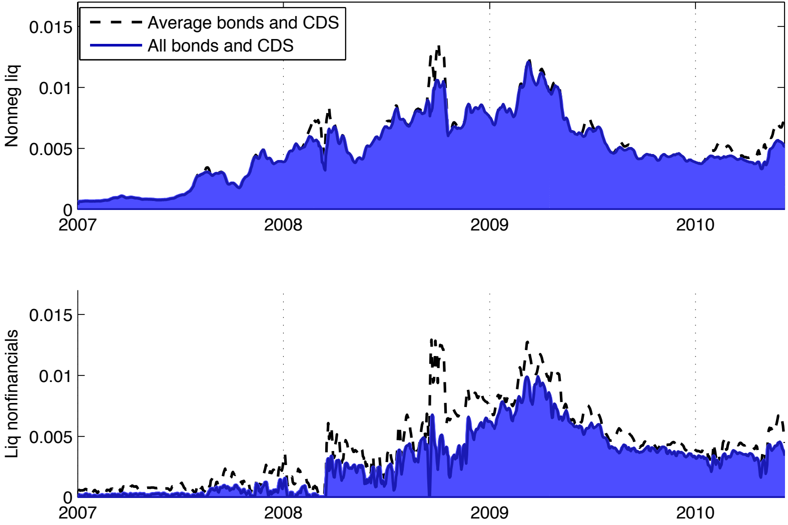
- p is *symmetric* if it does not depend on the ordering of A_i's.
- A LP problem

$$max\, c'p$$

$$s.t.\, Ap \leq b$$

  is *symmetric* if $c'p$ and all constraints do not depend on the ordering of A_i's.
- Example: union of all events, average of probabilities
- **Proposition 3:** problem is symmetric => $\exists$ symmetric solution
- **Corollary:** symmetric systems have the widest bounds given average probabilities

Symmetry



Average bonds and CDS
All bonds and CDS
Nonneg liq
0.015
0.01
0.005
0
2007
2008
2009
2010
Liq nonfinancials

Robustness: R and S

- **Dependence on R**
  - Two-period case:

$$p_i = 1 - (1 - R)P(A_i)$$

$$z_{ji} = (1 - R)P(A_i) - (1 - S)(1 - R)P(A_i \cap A_j)$$

  - Higher R $\Rightarrow$ lower yield $\Rightarrow$ bond-implied probability scales up
  - Higher R$\Rightarrow$ lower CDS spread $\Rightarrow$ cds-implied probabilities scale up
  - Bounds scale up

Robustness: R and S

- **Dependence on S**
  - Depends on whether the basis can be all explained by counterparty risk
  - Remember the constraints:

$$P(A_i) \leq a_i(\gamma_i)$$

$$P(A_i) - (1-S)\frac{\sum_{j \neq i} P(A_i \cap A_j)}{N-1} = \overline{p}_i(\overline{z}_i)$$

  - S=1 $\Rightarrow$ For each i, $P(A_i) = \overline{p}_i$
  - Decrease S $\Rightarrow P(A_i) > \overline{p}_i$: counterparty risk
  - But if S large, $P(A_i) - \overline{p}_i$ requires high counterparty risk

Robustness: R and S

$$P(A_i) \leq a_i(\gamma_i)$$

$$P(A_i) - (1-S)\frac{\sum_{i \neq j} P(A_i \cap A_j)}{N-1} = \overline{p}_i(\overline{z}_i)$$

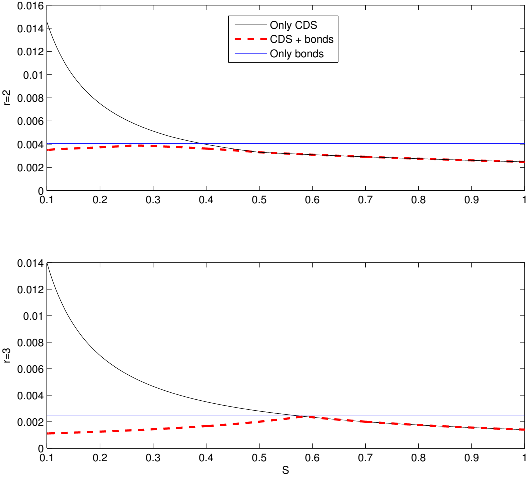
- S decreases more $\Rightarrow$ $P(A_i \cap A_j)$ can fill a larger gap $\Rightarrow$ systemic risk increases
- This *ignores* constraints from bonds
- Once $P(A_i)$ hits the upper bound $a_i(\gamma_i)$, $P(A_i \cap A_j)$ has to decrease

Robustness: R and S

- Example: June 25, 2008. Bank of America, Citigroup, GS. Probabilities are average monthly risk-neutral probabilities in bp.

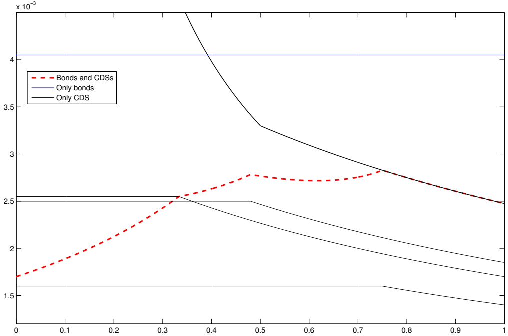
Bank	$a_i(0)$	$\overline{p}_i$
1	25	14
2	29	18.5
3	27	17

Robustness: R and S



Robustness: R and S

Bank	$a_i(0)$	$\overline{p}_i$
1	15(*not* 25)	14
2	29	18.5
3	27	17



Robustness: R and S

Model		Max P1					
R	S	2007	Jan 2008 to Bear	Bear to Lehman	Month after Lehman	Oct 2008 to April 2009	After April 2009
0.10	0.10	50.4	178.0	168.8	298.1	221.7	133.0
0.10	0.30	50.4	178.0	168.8	298.1	221.7	133.0
0.10	0.40	50.4	178.0	168.8	298.1	221.7	133.0
0.10	0.70	50.4	178.0	168.8	298.1	221.7	133.0
0.10	0.90	50.4	178.0	168.8	298.1	221.7	133.0
0.10	1.00	50.4	178.0	168.8	298.1	221.7	133.0
0.30	**0.30**	**64.8**	**228.9**	**217.0**	**383.3**	**285.0**	**171.1**
0.30	0.40	64.8	228.9	217.0	383.3	285.0	171.1
0.30	0.70	64.8	228.9	217.0	383.3	285.0	171.1
0.30	0.90	64.8	228.9	217.0	383.3	285.0	171.1
0.30	1.00	64.8	228.9	217.0	383.3	285.0	171.1
0.40	0.40	75.6	267.1	253.2	447.1	332.5	199.6
0.40	0.70	75.6	267.1	253.2	447.1	332.5	199.6
0.40	0.90	75.6	267.1	253.2	447.1	332.5	199.6
0.40	1.00	75.6	267.1	253.2	447.1	332.5	199.6

Robustness: R and S

Model		Max P4					
R	S	2007	Jan 2008 to Bear	Bear to Lehman	Month after Lehman	Oct 2008 to April 2009	After April 2009
0.10	0.10	2.2	2.3	15.7	24.3	49.1	30.9
0.10	0.30	2.4	2.5	17.3	25.7	48.8	31.2
0.10	0.40	2.5	2.6	18.2	26.2	48.6	31.2
0.10	0.70	2.8	2.9	20.0	27.7	47.2	30.6
0.10	0.90	3.3	3.3	21.5	28.4	46.0	29.7
0.10	1.00	12.6	44.5	42.2	70.0	55.4	33.3
0.30	**0.30**	**3.3**	**3.5**	**24.7**	**42.1**	**64.8**	**40.6**
0.30	0.40	3.4	3.7	25.6	42.5	64.2	40.5
0.30	0.70	3.7	4.1	27.9	43.7	62.0	39.6
0.30	0.90	4.1	4.6	29.5	44.3	60.6	38.3
0.30	1.00	16.2	57.2	54.3	90.0	71.2	42.8
0.40	0.40	4.0	4.4	32.0	54.0	76.8	47.5
0.40	0.70	4.3	5.0	34.7	54.0	74.0	46.3
0.40	0.90	4.8	5.7	36.1	53.3	72.0	44.8
0.40	1.00	18.9	66.8	63.3	104.9	83.1	49.9

Robustness: R and S

Model		Min P1					
R	S	2007	Jan 2008 to Bear	Bear to Lehman	Month after Lehman	Oct 2008 to April 2009	After April 2009
0.10	0.10	42.6	164.9	121.7	221.9	109.8	62.3
0.10	0.30	42.0	164.3	117.4	217.3	101.3	56.1
0.10	0.40	41.8	163.8	115.0	214.7	97.0	53.0
0.10	0.70	40.9	162.2	107.2	205.8	85.8	44.4
0.10	0.90	39.1	160.6	101.1	202.8	80.6	40.3
0.10	1.00	7.4	27.3	25.1	84.7	41.3	22.6
0.30	**0.30**	**53.9**	**210.3**	**146.4**	**252.2**	**125.9**	**71.3**
0.30	0.40	53.6	209.5	143.1	248.0	120.4	67.3
0.30	0.70	52.5	206.6	132.9	237.4	106.5	56.6
0.30	0.90	50.9	204.8	125.3	232.8	99.4	51.6
0.30	1.00	9.5	35.1	32.2	108.9	53.1	29.0
0.40	0.40	62.4	243.7	161.9	281.8	135.8	77.9
0.40	0.70	61.2	241.3	149.8	273.5	120.1	65.6
0.40	0.90	59.7	238.9	142.3	268.4	111.1	59.7
0.40	1.00	11.1	40.9	37.6	127.0	61.9	33.8

Time-varying recovery rates

- R could be lower in bad times
  - Adjusting $R \downarrow$ would imply bounds $\downarrow$
- S could be lower in bad times
  - lower S -> joint default risk has greater effect on basis
  - in peak episodes, basis is small -> joint default risk even smaller

Stochastic Recovery Rates

- Bonds and CDSs price in stochastic recovery rate
- Recovery rate depends on number of defaults
- Simple case: R_H if 1 bank defaults, R_L if more banks default
- Call $B(R_H, R_L)$ the price of a bond, $z(R_H, R_L)$ the price of a CDS

Stochastic Recovery Rates

- Show that:

$$B(R_H, R_L) = B(R_L, R_L) + Y_{bond}(R_{H,} R_L)$$

- And:

$$\sum_{s=1}^{T} \delta(0, s-1)(1 - P(A_i \cup A_j))^{s-1} z_{ji}(R_H, R_L) =$$

$$= \sum_{s=1}^{T} \delta(0, s-1)(1 - P(A_i \cup A_j))^{s-1} z_{ji}(R_L, R_L) - Y_{cds}(R_{H,} R_L)$$

- with $Y_{bond} \approx Y_{cds}$

Stochastic Recovery Rates

- Yields and CDS spreads are
  - Rescaled as if $R = R_L$
  - Shifted by a constant
- Adding Y to both bonds and CDSs does not change the basis
- The relevant rate is R_L

Other robustness tests

Model	Max P1					
	2007	Jan 2008 to Bear	Bear to Lehman	Month after Lehman	Oct 2008 to April 2009	After April 2009
Baseline	**64.8**	**228.9**	**217.0**	**383.3**	**285.0**	**171.1**
Using swap rates	64.8	229.0	217.1	383.6	285.2	171.1
US banks	49.4	166.7	156.7	278.7	182.7	102.2
US banks, larger trans	46.5	166.4	156.6	278.7	183.3	96.5
Reweight top 5 banks	65.0	228.9	217.0	383.3	285.8	171.3
Reweight, decreasing	65.0	228.9	217.0	383.3	285.8	171.3
Alternative bond model	35.2	112.0	162.2	579.0	216.9	51.9

Other robustness tests

Model	Max P4					
	2007	Jan 2008 to Bear	Bear to Lehman	Month after Lehman	Oct 2008 to April 2009	After April 2009
Baseline	**3.3**	**3.5**	**24.7**	**42.1**	**64.8**	**40.6**
Using swap rates	2.3	2.6	19.3	42.5	58.1	28.0
US banks	1.3	0.8	11.3	36.9	36.0	17.0
US banks, larger trans	1.4	1.0	16.6	39.6	42.1	16.6
Reweight top 5 banks	5.3	5.0	32.4	50.3	74.5	44.7
Reweight, decreasing	5.2	5.1	32.9	50.1	75.6	44.8
Alternative bond model	2.6	4.7	18.7	49.5	36.7	10.1

Other robustness tests

Model	Min P1					
	2007	Jan 2008 to Bear	Bear to Lehman	Month after Lehman	Oct 2008 to April 2009	After April 2009
Baseline	**53.9**	**210.3**	**146.4**	**252.2**	**125.9**	**71.3**
Using swap rates	56.6	217.0	156.1	258.2	141.5	97.8
US banks	43.3	157.9	117.0	187.0	110.6	62.3
US banks, larger trans	39.8	154.1	109.8	168.8	100.6	57.4
Reweight top 5 banks	50.4	204.8	128.2	243.5	121.5	67.5
Reweight, decreasing	50.9	204.4	128.2	249.7	121.1	64.6
Alternative bond model	25.2	92.4	107.5	397.8	121.3	26.6

Other robustness tests - derivations

- Alternative Pricing Model
- Using Swap Rates
- Different weighting in CDS
- Different currencies
- TRACE, larger transactions

Robustness: Pricing model

- Hazard rate deterministic but not constant:

$$h_{t+s} = (1-\rho_t)\overline{h}_t + \rho_t h_{t+s-1}$$

- CDS: assume joint default risk inherits ρ_t and $\overline{h_t}/h_t$ from reference entity

- Approximate around $h_t = 0$

Robustness: Swap rates

- Interest Rate Swaps
  - contain counterparty risk
  - are not indexed to a risk-free short rate
- Swap rates are higher than Treasuries -> lower systemic risk
- However, partly offset by calibrated liquidity process

Robustness: Weighting scheme

- If not all dealers post quotes every day, observed average will overrepresent more active banks
- Assume CDS spread is:

$$\bar{z}_i = \left[P(A_i) - (1-S) \left(\sum_{i \neq j} w_j P(A_i \cap A_j) \right) \right] \frac{\left[\sum_{s=t+1}^{T} \delta(t,s)\right](1-R)}{\left[\sum_{s=t}^{T-1} \delta(t,s)\right]}$$

with $w_j \neq \frac{1}{N-1}$

Robustness: Weighting scheme

- Obtain list of top 5 counterparties by trade count
- Two schemes (call w the weight of banks 6-15):
  1. $5w, 5w, 5w, 5w, 5w$
  2. $10w, 8w, 6w, 4w, 2w$

Robustness: Currencies

- Bonds and CDSs denominated in different currencies
- What assumptions do we need to mix them?
- Two bonds, same firms, different currencies
- $s = 0$ or i, default state
- e exchange rate
- m_{se} SDF
- Joint distribution of s and e

$$f(s,e) = \pi_s f_s(e)$$

Robustness: Currencies

$$p_i^{\$} = \pi_0 E[m_{se}|s=0] + R\pi_i E[m_{se}|s=i]$$
$$= E[m_{se}] - (1-R)\pi_i E[m_{se}|s=i]$$

$$p_i^E e_0 = E[e \cdot m_{se}] - (1-R)\pi_i E[e \cdot m_{se}|s=i]$$
$$t^{\$} = E[m_{se}]$$
$$t^E e_0 = E[e \cdot m_{se}]$$

Robustness: Currencies

$$P(A_i) = \pi_i \frac{E[m_{se}|s=i]}{E[m_{se}]}$$

From Euro bonds, we obtain

$$\pi_i \frac{E[m_{se}|s=i]}{E[m_{se}]}$$

So we can mix if:

$$\frac{E[e \cdot m_{se}|s=i]}{E[m_{se}|s=i]} = \frac{E[e \cdot m_{se}]}{E[m_{se}]}$$

Robustness: Currencies

- Now take Euro-denominated CDS for i. Counterparty j American.
- $s \in \{i, j, ij, 0\}$

$$z_{ji} e_0 = (1-R)\pi_i E\left[e \cdot m_{se} | s = i\right] + (1-R) S \pi_{ij} E\left[e \cdot m_{se} | s = ij\right]$$

$$= E[e \cdot m_{se}] \left((1-R)\pi_i \frac{E\left[e \cdot m_{se} | s = i\right]}{E[e \cdot m_{se}]} + (1-R) S \pi_{ij} \frac{E\left[e \cdot m_{se} | s = ij\right]}{E[e \cdot m_{se}]} \right)$$

- So: condition is for every s

$$\frac{E[e \cdot m_{se} | s]}{E[m_{se} | s]} = \frac{E[e \cdot m_{se}]}{E[m_{se}]}$$

Robustness: TRACE, larger transactions

- Quoted data might have lags and matrix prices
- Small trades might be less reflective of credit risk
- Results using
  - only US banks
  - TRACE trades >= $100,000

Discussion: Arora et al.

- Arora, Gandhi and Longstaff (2010) run the regression for bond k:

$$z_{jk,t} = a_{k,t} + bz_{j,t-1} + e_{jk,t}$$

  - Find that b is negative but small
  - Default probability of the counterparty little cross-sectional effect on price.
- First point:
  - The starting point of my paper is the difference between $P(A_j)$ and $P(A_i \cap A_j)$
  - Only $P(A_i \cap A_j)$ is priced in the CDS, not $P(A_j) \approx z_{j,t}$.
- Second point:
  - $a_{k,t}$ removes all *average* counterparty risk
  - This paper is based *only* on the pricing of average counterparty risk
  - Even if for some reason there is compression of quotes

Discussion: Arora et al.

- Third point:
  - cross-sectional difference in S (collateralization) might induce lower dispersion of quotes
  - If S_j is different by j the average quote reflectes a *weighted average* of $P(A_i \cap A_j)$

$$\frac{z_{1i}+z_{2i}}{2} = P(A_i) - \frac{(1-S_1)}{2} P(A_i \cap A_1) - \frac{(1-S_2)}{2} P(A_i \cap A_2)$$

$$= P(A_i) - (1-S)[\frac{(1-S_1)}{(1-S)2} P(A_i \cap A_1) + \frac{(1-S_2)}{(1-S)2} P(A_i \cap A_2)]$$

where $S = \frac{S_1+S_2}{2}$

  - Lower collateral requirement -> lower S -> higher weight
  - Robustness: biggest dealers (Goldman, DB, JPM) safer -> less collateral
  - Smaller dealers (Lehman, Merrill) -> more collateral

Explore: ETF binary puts